Module 2 Review sheet

1. What are forecast origin, forecast horizon?

Forecast origin: the point in time from which the forecasts start.
Forecast horizon: how far ahead do we wish to forecast.

Example:
$F_{13}(2)$: forecast horizon: 2
forecasting origin: 13 – 2 = 11

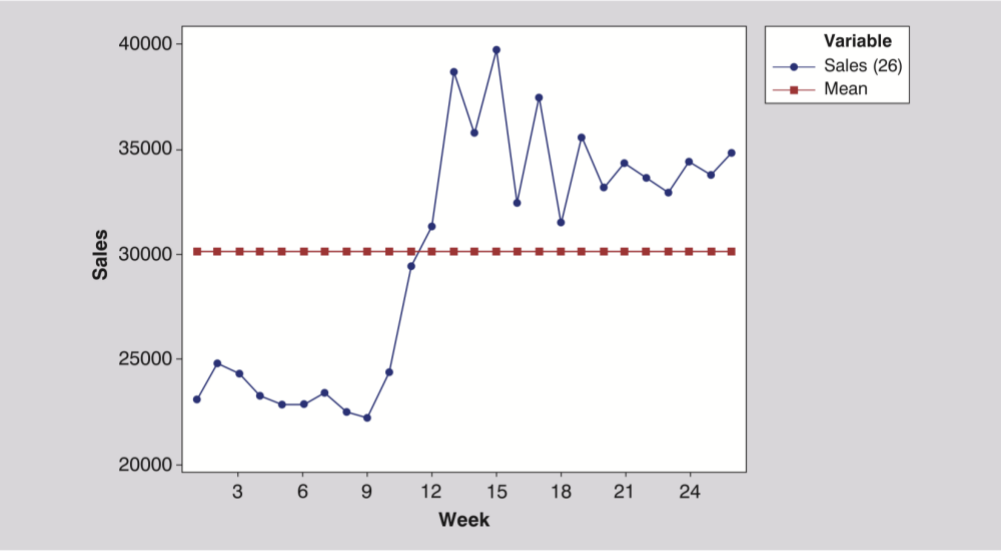
Table 3.1 Notation for Forecasts Made at Different Forecast Origins and for Varying Forecast Horizons

Forecast Origin	12	13	14
Forecast for period 13	$F_{13}(1)$		
Forecast for period 14	$F_{14}(2)$	$F_{14}(1)$	
Forecast for period 15	$F_{15}(3)$	$F_{15}(2)$	$F_{15}(1)$

2. What is Locally constant forecasts? What does the time series plot look like?

$$F_{t+h}(h) = \text{constant}$$

The plot showing future forecasts is a horizontal line.



3. What is moving average? How to calculate the MA(t|K)? How to use Moving Average to make prediction?

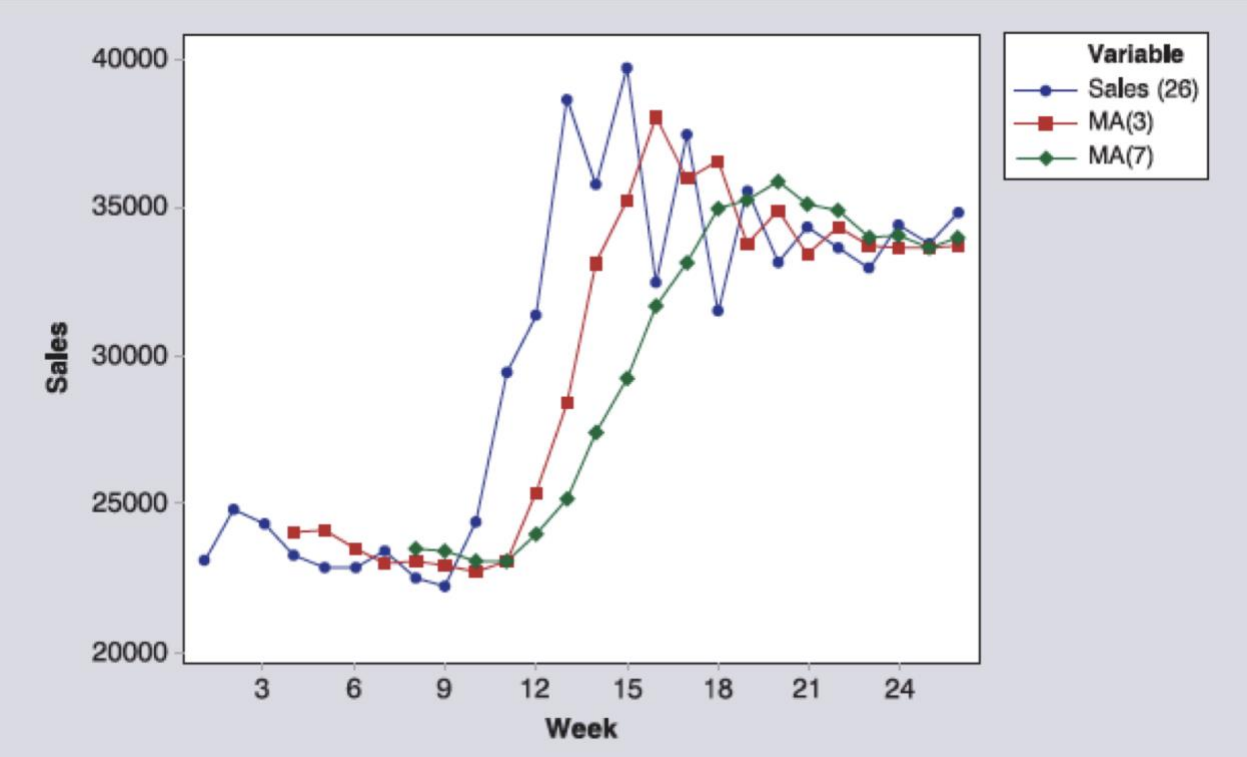
the moving average of order K evaluated at time t is denoted by MA(t|K), which is the average of last K terms of the series:

$$MA(t|k) = \frac{Y_t + Y_{t-1} + \cdots + Y_{t-K+1}}{K}$$

We can forecast the time series at time (t+1) using MA(t|K):

$$F_{t+1} = MA(t|K)$$

4. Larger values of K produce smoother plots but are slower to adapt to changes.



5. What is centered moving average? How to manually calculate CMA(K)?

When K is odd: CMA(k) is the average of

$$Y_{t-\frac{k-1}{2}}, \ \ldots, Y_t, \ldots, Y_{t+\frac{k-1}{2}}$$

When K is even:

$CMA(K) = (MA_t(K) + MA_{t+1}(K))/2$ and placed at period $t–k/2+1$

K = 4, t = 4 CMA(4)

CMA(3.5) = mean(Y_2, Y_3, Y_4, Y_5)
CMA(4.5) = mean(Y_3, Y_4, Y_5, Y_6)

6. What is a Simple Exponential Smoothing? What is the Error Correction form:

$$F_{t+1} = L_{t+1} = L_t + \alpha e_t$$

F_{t+1} is the 1-step ahead prediction for time t+1.
e_t is the 1-step ahead prediction error for time t. $e_t = Y_t - F_t$
α is the smoothing parameter used to update the local level term.

Smaller α gives slow adjustment. Larger α gives rapid adjustment.
$\alpha = 0$ means there is a fixed level term.

7. What is the 1-step ahead forecast using SES? What is the h-step ahead forecast using SES?

$$F_{t+h}(h) = F_{t+1}(1) = F_{t+1} = L_{t+1} = L_t + \alpha e_t$$

The h-step ahead forecast is the same as the 1-step ahead forecast using SES.

8. What is a Linear Exponential Smoothing? How do we update the level term and the trend term?

Forecast Error:

$$e_t = Y_t - F_t = Y_t - (L_{t-1} + T_{t-1})$$

Update the level term and trend term:

$$L_t = L_{t-1} + T_{t-1} + \alpha e_t$$

$$T_t = T_{t-1} + \alpha\beta e_t$$

The h-step ahead forecast:

$$F_{t+h}(h) = L_t + hT_t$$

α is the smoothing parameter used to update the local level term.
β is the smoothing parameter used to update the local trend term.

9. What is the 1-step ahead forecast using LES? What is the h-step ahead forecast using LES?

1-step ahead forecast:

$$F_{t+1}(1) = L_t + T_t$$

h-step ahead forecast:

$$F_{t+h}(h) = L_t + hT_t$$

10. When should we use Holt-Winters Model?

Holt- Winters Model includes level term, trend term and seasonal components.

11. How do we update the level, trend, and season term for Holt-Winters Model?

$$L_t = L_{t-1} + T_{t-1} + \alpha e_t$$

$$T_t = T_{t-1} + \alpha\beta e_t$$

$$S_t = S_{t-m} + \gamma e_t$$

12. Meaning of smoothing parameters of a Holt-Winters Model. Special cases when certain smoothing parameters are 0.

α is smoothing parameter for level term.
β is smoothing parameter for trend term.
γ is smoothing parameter for seasonal component.

Special Cases:

- Fixed seasonal pattern: $\gamma = 0$ (no seasonal updating)
- No seasonal pattern: $\gamma = 0$ and all initial S values are set equal to zero
- Fixed trend: $\beta = 0$
- Zero trend: $\beta = 0$ and $T_0 = 0$
- All fixed components: $\alpha = \beta = \gamma = 0$

13. What is a State Space Model? What are observation equations? What are state equations?

A state space model consists of two parts:
1. An observation equation that relates the random variables (Yt, the actual value) to the underlying state variable(s).
2. One or more state equations that describe how the state variables evolve over time.

14. What are state variables for SES, LES, Holt-Winters Model?

SES: L
LES: L, and T
Holt-Winters Model: L, T, and S

15. What are observation equation and state equation for SES?

Observation equation: $Y_t = L_{t-1} + \epsilon_t$
State equation: $L_t = L_{t-1} + \alpha\epsilon_t$
Forecast function: $F_t = L_{t-1}$

16. What are observation equation and state equation for LES?

Observation equation: $Y_t = L_{t-1} + T_{t-1} + \epsilon_t$

State equation: $L_t = L_{t-1} + T_{t-1} + \alpha\epsilon_t$
$T_t = T_{t-1} + \alpha\beta\epsilon_t$

Forecast function: $F_{t+h}(h) = L_{t-1} + hT_{t-1}$

17. What are observation equation and state equation for Additive Holt-Winters Model?

Observation equation: $Y_t = L_{t-1} + T_{t-1} + S_{t-m} + \epsilon_t$

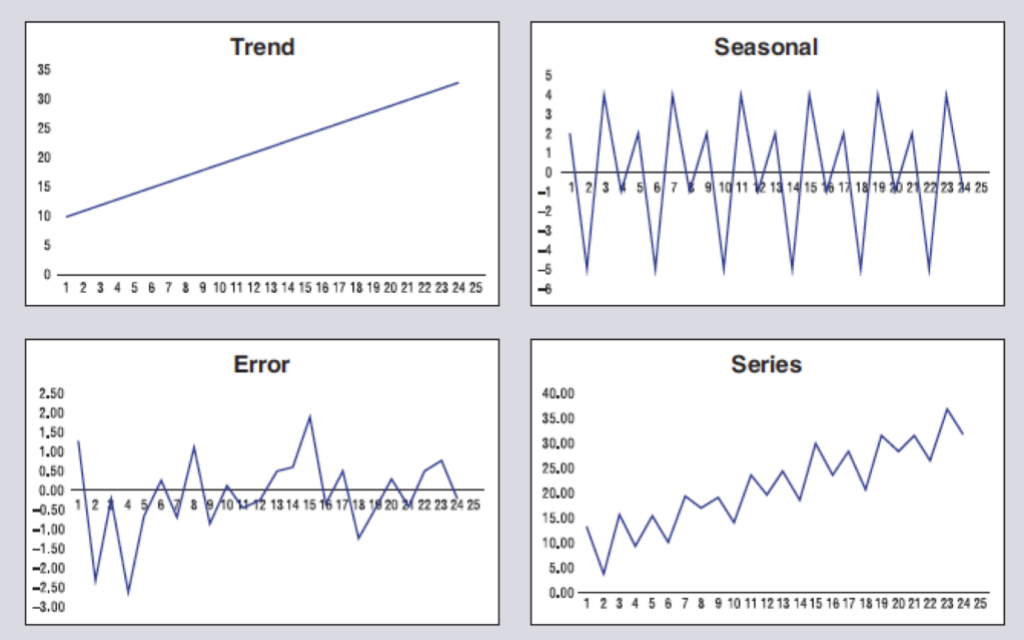
State equation: $L_t = L_{t-1} + T_{t-1} + \alpha\epsilon_t$
$T_t = T_{t-1} + \alpha\beta\epsilon_t$
$S_t = S_{t-m} + \gamma\epsilon_t$

Forecast function: $F_{t+h}(h) = L_{t-1} + hT_{t-1} + S_{t+h-m}$

18. What are the 3 components of time series data?

T = Trend + Cycle
S = Seasonal Components
E = Random Error

19. Time series plot of original time series data and time series plot of each component



20. What are the relationship between the actual values (Y) and the 3 components in different models?

Additive model: Y = T + S + E
Multiplicative model: Y = TSE
Mixed additive-multiplicative model: Y = TS + E; or Y = (T + S)E

21. How to use R to fit a State Space Model? How to make point prediction and construct a prediction interval?

Method	Observation Equation	ETS
SES	Y = L + E	A,N,N
LES	Y = L + T + E	A,A,N
Multiplicative LES	Y = L * T * E	M,M,N
Seasonal SES	Y = S + E	A, N, A
Additive Holt-Winters	Y = L + T + S + E	A, A, A
Multiplicative Holt-Winters	Y = (L + T)* S * E	M, A, M

Example:
Suppose we have a time series object from time period 1 to time period t : data.ts
We want to fit an additive Holt-Winters Model

R code:

```
# Fit an additive Holt-Winters Model
    model <- ets( data.ts, model = "AAA", damped = False)
# Make 1-step ahead prediction for time period t+1
    frc1 <- forecast(model, h = 1)
# Point estimation and prediction interval
    frc1

# Make prediction for time period t+1, t+2, …, t+m
    frc_m <- forecast(model, h = m)
    frc_m
```